Push or Pull? Grants, Prizes and Information *

David Rietzke†

August 21, 2015

Abstract

I study how a research funder optimally encourages a researcher to undertake R&D activity in a setting where: the researcher is privately informed about the prospects of the project, and R&D input consists of both observable and non observable components. In the presence of adverse selection and moral hazard, push programs – which reward researchers independently of their output – have been criticized since they may: (1) finance projects unlikely to succeed, and (2) provide weak incentives for unobservable inputs. Despite these criticisms, I find that a push program may emerge as the optimal means of funding as a consequence of the interaction between adverse selection and moral hazard.

KEYWORDS: Grants, Prizes, Moral Hazard, Adverse Selection, Innovation, Observable Input, Principal-Agent Problem
JEL Classifications: D82, D86, O31

*I thank Stan Reynolds, for his valuable input throughout this project. I also thank Asaf Plan, John Wooders, Andreas Blume, Martin Dufwenberg, Rabah Amir, Derek Lemoine, Yu Chen, Dakshina De Silva, Rachel Pownall and Tim Flannery for helpful comments and suggestions.

†Department of Economics, Lancaster University, d.rietzke@lancaster.ac.uk

1 Introduction

Innovation drives long-run economic growth, but in many instances the social value of an innovation exceeds the private value to the innovator. As a result, investment in R&D may fall well below the socially optimal level.[1] The tools used to encourage R&D activity can be categorized as either push or pull mechanisms. Push mechanisms – such as research grants, tax credits on R&D, or direct inputs from a funder – operate upstream and subsidize research inputs. Pull mechanisms – such as innovation prizes, tax credits on sales, or patent buyouts – operate downstream and reward successful research output. Research grants and innovation prizes are commonly used by both public and private agencies to encourage R&D activity. The Bill and Melinda Gates Foundation, for example, issued more than $2.6 billion in grants in 2012.[2] Innovation prizes are a key component of the Obama Administration's attempt to stimulate American innovation.[3] From 2010 to 2012, 200 new innovation prizes were offered by federal agencies in areas ranging from national defense to education.[4]

Research funders often contend with two informational asymmetries: (1) a researcher may be better informed about the prospects of a given project (adverse selection; AS), and (2) some relevant inputs may not be observable (moral hazard; MH). Given the AS and MH problems, push mechanisms – which reward a researcher independently of her output – have been criticized since they may (i) finance research with a small chance of success, and (ii) provide weak incentives for unobservable inputs. Pull mechanisms only reward successful research output; for this reason, it has been argued that they are better suited to deal with both AS and MH (Kremer, 2002).[5] Despite this

[1] Jones and Williams (1998) estimate that the socially optimal level of R&D spending is 2 to 4 times greater than observed investment. For a recent survey of the empirical literature on returns to R&D see Hall et al. (2009)

[2] http://www.gatesfoundation.org/Who-We-Are/General-Information/Financials

[3] http://challenge.gov/about

[4] http://www.whitehouse.gov/blog/2012/09/05/challengegov-two-years-and-200-prizes-later

[5] That compensation be tied to output in settings with moral hazard dates back to the work of Arrow (1970), Spence and Zeckhauser (1971), and Stiglitz (1974) among many

criticisms, I find that a push program may emerge as the optimal means of funding as a consequence of the interaction between AS and MH.

I study a model in which a researcher (she) expends costly research inputs to increase the likelihood that she develops a new technology; all else equal, a researcher of higher ability is more likely to innovate. Successful innovation generates profit for the researcher, but this incentive is insufficient to warrant R&D activity. A research funder (he) values the innovation, and incentivizes the researcher to undertake R&D. Under limited liability and a free-disposal condition, the optimal mechanism takes the form of a push and/or pull program. The push incentive – a *grant* – is a reward received by the researcher independently of whether she successfully innovates; the pull incentive – a *prize* – is a reward received by the researcher only if she succeeds. I wish to focus solely only on the incentive properties of push and pull mechanisms, so to abstract from risk-sharing considerations, the researcher and the funder are assumed risk neutral.

The researcher knows her own ability (type), but the funder knows only the distribution over types; the funder also faces a hidden action problem. It is easy to imagine that the time, effort, energy, etc. a researcher devotes to a project is prohibitively costly to quantify and/or verify. Yet some R&D inputs, such as large-scale capital investments (e.g. a new research facility), may be more easily observed. Indeed, there are many examples of funding programs that offer rewards dependent on R&D expenditure. Matching grants, for example, require investments from a recipient in excess of the value of the grant. The U.S. R&D-tax-credit system offers credits that are proportional to a firm's qualifying R&D expenditures. Some credits take the form of push programs, rewarding a firm simply for engaging in R&D activity, while others take the form of pull programs, requiring tangible results.[6] In this spirit, I consider an

others. Hölmstrom (1979) observed that contracts should depend on any observable signal that provides information about the level of unobserved effort.

[6]One example of such a pull program may be found in the U.S. "Orphan Drug Act" of 1983, which offers a tax credit – equal to half the clinical cost of development – for the successful creation of certain new pharmaceuticals (Reider, 2000). See also Hemel and Ouellette (2013) for a comprehensive overview of the U.S. R&D tax credit system.

environment with two research inputs. One input – *effort* – is not observable; the other input – *investment* – is observable. Investment and effort are both essential for the success of the project, and are strategic complements.

I show that a grant may emerge as the optimal means of funding as a result of the interaction between AS and MH.[7] To convey the basic intuition, first consider a pure AS environment (i.e. all relevant inputs are observable, but the researcher's ability is unknown to the funder). In this setting, the optimal means of funding offers no prize, and takes the form of a matching grant. Intuitively, the researcher must be permitted to capture at least as much rent as she could by imitating a lower type; this rent is proportional the lower type's reward for innovation. A prize, therefore, would generate greater information rents for the researcher, and is a more costly means of funding from the funder's perspective. The matching feature of the grant contract deters a less-able researcher from exaggerating her type; hence, the funder avoids awarding a large grant to a researcher with a small chance of success.

In a pure MH environment (i.e. some relevant research inputs are non observable, but the researcher's ability is known), the optimal means of funding, in general, uses only a prize. The prize creates a naturally strong incentive for the researcher to exert unobservable effort, since it is only received if the project succeeds. It is worth pointing out, however, that a grant can be used to provide incentives for unobservable effort. The grant can be used to elicit a larger investment from the researcher, which increases the marginal returns to effort. As there is some profit incentive associated with innovation, this leads the researcher to exert greater effort, provided the MH problem is not too severe. In some cases, a pure grant can even emerge as an optimal means of funding.

When AS and MH interact, the optimal means of funding typically involves the use of both a prize and a grant. The funder faces a familiar trade-off between providing strong incentives for effort, and limiting the information rents that accrue to the researcher; a prize is more efficient in dealing with the

[7] More precisely, when referring to "a grant" or "a prize" in the context of the optimal contract under AS, I mean a menu of grants/prizes.

former, while the grant is more efficient in dealing with the latter. In some circumstances a matching grant emerges as the optimal means of funding. These circumstances can be interpreted as a setting in which the researcher has a strong incentive to devote her time to a project, but due to frictions in the financial sector, is unwilling to acquire the necessary capital. Interestingly, within this environment, as the researcher's cost of effort increases (translating to a more severe moral hazard problem), prizes, in some sense, become even less attractive as compared to grants. The reason is that the MH problem exacerbates the AS problem, which makes grants relatively more attractive. Finally, in some cases a prize may be necessary to elicit R&D activity. Yet, even in such an environment, the optimal means of funding still may involve the use a grant, as a way to limit the researcher's information rents.

The contributions of this paper are twofold. First, my results add to the literature on innovation incentives. I provide a justification for the the use of push programs in the presence of AS and MH, and deliver recommendations for how push programs may cope with these informational asymmetries. Second, this paper contributes to the literature on optimal contracting; specifically, my findings deliver new insights on input and output-based rewards. I discuss in greater detail my contribution and related work in the next section.

2 Related Literature

Optimal Contracting

Central to my results is a complementarity between the observable and non observable input. Other authors have explored the relationship between the power of incentive contracts, and complementarities between inputs. Most notably is a *crowding-out effect* that may arise when an agent concurrently undertakes multiple tasks (Holmstrom and Milgrom, 1991; Laffont and Tirole, 1993, ch. 4). The essence of the crowding-out effect is that a powerful incentive scheme for one task, may deter the agent from undertaking another task, if those two tasks are strategic substitutes (i.e. if the marginal cost of effort for one task increases in the level of effort devoted to another task). My focus is not on the power of incentive schemes, but rather the circumstances under which it is optimal to base rewards solely on an observable input, and when it is optimal to also condition rewards on output.[8] I find that, although output serves as an informative signal for effort, it may be optimal for the principal to ignore this signal. Holmstrom and Milgrom show that the crowding-out effect may also lead the principal to ignore an informative signal, and in some cases offer a "fixed-wage contract"; my finding differs in two ways. First of all, there is a fundamental difference between the fixed-wage contract discussed by Holmstrom and Milgrom and the grant in my setting. The fixed wage is a reward received by the agent independent from any signal the principal observes; the grant is a reward that depends on the researcher's investment, but is independent from output. Second, the crowding-out effect relies on a strategic substitutability, while my results rely on a strategic complementarity, and the interaction between MH and AS.[9]

Holmstrom and Milgrom's model can accommodate observable actions, but their focus is primarily on settings with multiple unobservable actions. To the best of my knowledge, little work has been done on contracting models with both observable and unobservable actions. In another context, Choi (1992) shows that free-riding incentives in a research joint venture can be alleviated if there is another complementary and contractible input. Independent work by Chen (2013) studies the inclusion of observable actions in contracting models with MH, under quite general conditions. The author establishes existence of optimal contracts, and provide sufficient conditions under which contracts contingent on the observable action strictly increase the principal's payoff. My model trades off generality for more precise statements about the form of optimal contracts, and is specifically tailored to obtain insights on the funding

[8]In my context, comparing the "power" of incentive schemes can lead to ambiguity; it is much more meaningful to distinguish between input-based and output-based rewards, a point also made by Lazear (2000).

[9]Laffont and Tirole's model does have both an AS and MH problem, but it is still the crowding-out effect – driven by a strategic substitutability between actions – that drives their results.

of R&D. Moreover, Chen (2013) does not study the interaction of AS and MH, which is of central focus in this paper.

It is also worth noting how optimal contracts, in the presence of observable and non observable inputs, contrast similar models with only unobservable input(s) (see, for example, Lewis and Sappington, 2000a,b, 2001). The most obvious point of contrast is the ability of output-independent transfers to encourage non-observable inputs. In the settings considered by Lewis and Sappington, such transfers are completely ineffective.[10] The presence of an observable input means that transfers need not depend on output to provide strong incentives for non-observable inputs. Second, Lewis and Sappington (2000b) show that if agent-to-principal transfers are not feasible, then in the presence of AS and MH the principal loses the ability to tailor rewards to different types, and pooling is necessary.[11] The presence of a contractible input provides the principal with another tool that can be used to sort over types, thus avoiding the necessity to pool.

Innovation Incentives

Maurer and Scotchmer (2003) offer an explanation for how grants might overcome MH, and make the point that matching grants can be an effective screening device in the presence of AS. As regards MH, the authors posit that repeated interaction between researchers and funders instills a discipline in the researchers; researchers that fail to deliver in the past are denied future grants. My results provide an additional channel – which is relevant in a static setting – through which grants may help overcome MH. My results confirm the intuition of Maurer and Scotchmer as regards the effectiveness of matching grants in dealing with AS. Yet my analysis – which relies on a mechanism design approach – reveals the circumstances under which a matching grant may be the *optimal* means of funding in the presence of both AS and MH.

[10] Of course, if the agent is risk-averse then output-independent rewards may arise as a risk-sharing device; but my focus is not on optimal risk sharing.

[11] Lewis and Sappington show that this issue can be overcome if there are multiple agents. For a recent examination of how simultaneous AS/MH can lead to pooling in a similar model, see Gottlieb and Moreira (2014, 2015).

Prizes have received considerable attention in the literature on innovation, with particular attention paid to informational trade-offs between prizes and intellectual property rights (e.g. patents);[12] fewer studies have provided a direct comparison of the incentive properties of push and pull programs.[13] One notable exception, is Fu et al. (2012) (henceforth, FLL). In FLL's model, a budget-constrained funder allocates her budget between a prize and a subsidy to each of two researchers engaged in a patent race. The prize is a reward received by the first researcher to innovate, while the subsidy is a direct input that increases the productivity of the researcher's effort (Lewis and Sappington, 2000b, also consider direct inputs from the principal). The authors identify trade offs between prizes and subsidies that are driven by their effect on the nature of competition. My analysis complements the work of FLL; I abstract from competition,[14] but highlight the informational trade-offs between push and pull incentives.

3 The Model

There are two players: the funder and the researcher. The funder may be thought of as a private NGO or a government agency. The researcher could be a for-profit firm, an academic researcher, or a nonprofit research foundation. The funder values the development of a new technology and offers incentives to the researcher to encourage costly R&D activity. Successful innovation requires both observable and non-observable inputs. The observable input – *investment* – may be thought of as a capital investment such as the purchase

[12]See, for example, Wright (1983), De Laat (1997), Scotchmer (1999), Hopenhayn et al. (2006), Chari et al. (2012), and Weyl and Tirole (2012)

[13]A similar question has been studied in the labor literature. Lazear (1986), for example, studies the trade-offs between salaries and piece rates. In his model, the salary encourages effort through the manager's ability to fire a worker, if her effort falls below some threshold level.

[14]One can think of a two-stage game where, in the first stage, the principal selects a particular researcher from a pool via some selection mechanism. In the second stage, funding negotiations take place. I abstract from stage 1 and focus on stage 2. Such a structure is not uncommon in practice. The Bill and Melinda Gates Foundation, for example, uses a similar process to select many of its grantees `http://www.gatesfoundation.org/How-We-Work`.

of new lab equipment, or the construction of a new research facility. The unobservable input –*effort* – may be thought of as the time or energy devoted by the researcher to the success of the project.

Denote the researcher's investment by $x \geq 0$, and her effort by $y \geq 0$. Given, $\theta \in (0, 1]$, the probability of successful innovation is $\theta\alpha(x, y)$. The function, $\alpha : \mathbb{R}^2_+ \to [0, 1]$, is twice continuously differentiable, increasing in both arguments, and strictly concave in both arguments. The two research inputs are strategic complements, so that higher levels of investment increase the marginal returns from effort (and vice versa). The researcher's technology is summarized as follows:

Technology.

T1: $\alpha(0, y) = 0$ *and* $\alpha(x, 0) = 0$ *for all* (x, y)

T1: α *is twice continuously differentiable, and for all* $(x, y) \in \mathbb{R}^2_{++}$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_{11} < 0$, $\alpha_{22} < 0$ *and* $\alpha_{12} > 0$.[15]

The marginal cost of investment is normalized to unity, while the marginal cost of effort is $c \geq 0$. I allow for an upper bound on the researcher's effort $\overline{y} \in (0, \infty]$. A finite upper bound is natural if, for example, one interprets effort as the fraction of the researcher's time devoted to the project. Innovation generates a profit, $\pi > 0$, for the researcher, and a payoff, $W > \pi$, for the funder. W might represent, for example, the benefit to society that is not captured by the firm. The researcher's profit, π, is insufficient to warrant R&D activity, but the total benefit of innovation, $W + \pi$, is sufficiently large to ensure positive gains from R&D activity. The next two conditions summarize the conditions given on the payoffs for innovation, π, and W:

Innovation Payoffs.

P1: $\overline{\theta}\alpha_1(0, \overline{y})\pi - 1 < 0$.

P2: $\max_{x,y} \{\underline{\theta}\alpha(x, y)(W + \pi) - x - cy\} > 0$.

[15]For a function, $H(x_1, \cdots, x_m)$, H_k denotes $\frac{\partial H(x_1, \cdots, x_m)}{\partial x_k}$

P1 says that, absent any other incentive, the researcher's payoff is strictly decreasing in her investment level, for any $y \in [0, \overline{y}]$. Condition *P2* ensures that there are gains to R&D activity from a researcher of any type. The parameter $\theta \in (0, 1]$ summarizes the private information held by the researcher about the likelihood the project will succeed. This information might reflect either the researcher's ability or the difficulty of the project; for concreteness, I adopt the former interpretation, and I use the terms *ability* and *type* interchangeably. The researcher's type, θ, is drawn from a continuous distribution with compact support, $\Theta \equiv [\underline{\theta}, \overline{\theta}] \subset [0, 1]$, according to CDF, F, and corresponding PDF f. The researcher knows the true θ while the funder knows only its distribution. To avoid two simultaneous AS problems, I assume that π is known by the funder.

3.1 The Funder's Problem

It suffices to consider only the use of direct mechanisms: first, the researcher announces her type, θ. Next, the funder makes a take-it-or-leave-it offer consisting of: an investment level, $x(\theta) \geq 0$; a transfer to the researcher the project fails, $t_f(\theta)$; and a transfer to the researcher if the project succeeds, $t_s(\theta)$.[16] Following Innes (1990) I assume the researcher is protected by limited liability: $t_f \geq 0$. After the investment contract is formed, the researcher chooses her effort level, $y \geq 0$, optimally. The outcome of the project is then realized, and the researcher reports success or failure to the funder, who may then investigate the researcher's claim. If the researcher reports success, I assume that the funder can verify or disprove the claim. Should the researcher succeed, I assume that she may shroud her success from the funder.[17] To prevent shrouding, the reward for success must be no less than the reward for failure: $t_s \geq t_f$. Under risk neutrality, the condition $t_s \geq t_f \geq 0$ implies that the mechanism is equivalent to one in which the funder offers a grant, $g \equiv t_f \geq 0$, and/or a prize

[16] $t_k(\theta) < 0$ represents a transfer from the researcher to the funder.

[17] Prior to the funder's inspection, the researcher could, for example, sabotage her own project to give the funder the appearance of failure, while still earning her private profit associated with success (π). Innes considers a similar condition.

$V \equiv t_s - t_f$; the grant is received regardless of whether the project succeeds, and the prize is earned only if the project succeeds.[18]

Now, if a type θ researcher invests x, and the reward for successful innovation is $V + \pi$, she chooses the effort level:

$$y(\theta, x, V + \pi) \equiv \arg \max_{y \in [0, \overline{y}]} \{\theta \alpha(x, y)(V + \pi) - cy\}$$

Note that for a given x and V, the researcher's effort problem is strictly concave in y, which ensures $y(\theta, x, V + \pi)$ is a single-valued function. The payoff to a researcher of type θ, who reports her type as $\hat{\theta}$, is given by:

$$U(\hat{\theta}|\theta) = \theta \alpha \left(x(\hat{\theta}), y(\theta, x(\hat{\theta}), V(\hat{\theta}) + \pi) \right) \left[\pi + V(\hat{\theta}) \right] - x(\hat{\theta}) - cy(\theta, x(\hat{\theta}), V(\hat{\theta}) + \pi) + g(\hat{\theta})$$

The funder's problem is given by:[19]

$$\max_{\{x(\theta), V(\theta), g(\theta)\}} \int_{\Theta} \left[\theta \alpha \big(x(\theta), y(\theta, x(\theta), V(\theta) + \pi) \big) \left[W - V(\theta) \right] - g(\theta) \right] dF(\theta) \quad (1)$$

Subject to individual rationality (IR), incentive compatibility (IC), and the non-negativity constraints: $x(\cdot) \geq 0$, $g(\cdot) \geq 0$, and $V(\cdot) \geq 0$. IR and IC require that for all θ, $\hat{\theta} \in \Theta$:

$$\text{(IR)} \quad U(\theta) \geq 0 \quad (2)$$

and

[18] One could envision an alternative set-up in which the principal does not mandate an investment level, but designs a reward structure that is dependent on observed investment: $g(\theta, x)$ and $V(\theta, x)$. The researcher then chooses x and y optimally according to these rewards. I lose no generality in my setup. It is straightforward to show that for any $g(\theta, x)$ and $V(\theta, x)$ the induced actions, (x^*, y^*) can be implemented via my mechanism.

[19] Note that the researcher's profit does not enter his objective function. The qualitative nature of my results generalize immediately to a specification in which the funder weights the researcher's profit by some constant, $a < 1$. This would capture, for example, an environment in which the funder's resources are obtained through socially costly taxation, or costly fundraising activities.

$$\text{(IC)} \quad U(\theta) \geq U(\hat{\theta}|\theta) \tag{3}$$

Where $U(\theta) \equiv U(\theta|\theta)$ is the researcher's payoff when she reports her type truthfully. Note that condition *P1* implies that the researcher's outside option is zero; this ensures that the IR constraint only requires that the researcher earn a non negative payoff. The IC constraint requires that truth-telling is optimal for the researcher.

3.2 Efficiency and Perfect Information

It is useful to have a notion of input levels that are socially efficient, and those that are efficient from the funder's perspective. This leads to the following two definitions.

Definition 1. *(Efficiency)*
Let $\mathcal{S}(\theta, x, y) \equiv \theta\alpha(x, y)(W + \pi) - x - cy$. The investment schedule, $x^e(\theta)$, and effort schedule, $y^e(\theta)$, are efficient if

$$\big(x^e(\cdot), y^e(\cdot)\big) = \arg \max_{x(\theta), y(\theta)} \{E_\theta[\mathcal{S}(\theta, x(\theta), y(\theta))]\} \quad s.t. \;\; \forall\theta \in [\underline{\theta}, \overline{\theta}] : x(\theta) \geq 0 \;\; and \;\; y(\theta) \in [0, \overline{y}]$$

$E_\theta[\mathcal{S}(\theta, x(\theta), y(\theta))]$ is the ex-ante expected total surplus that is generated when the researcher invests according to $x(\theta)$ and chooses effort according to $y(\theta)$. The efficient investment/effort schedules maximize expected total surplus. Note that, at each θ, $x^e(\theta)$ is characterized by the first-order condition:

$$\theta\alpha_1(x^e(\theta), y^e(\theta))(W + \pi) - 1 = 0$$

When interior at any θ, $y^e(\theta)$ satisfies:

$$\theta\alpha_2(x^e(\theta), y^e(\theta))(W + \pi) - c = 0$$

Note that $y^e(\theta) = \overline{y}$ if and only if $\theta\alpha_2(x^e(\theta), \overline{y})(W + \pi) - c > 0$. The next definition concerns mechanisms that are efficient from the perspective of the funder.

Definition 2. *(First Best)*
A mechanism, $\mathcal{M}$, attains the first-best for the funder if his equilibrium payoff is equal to $E[\mathcal{S}(\theta, x^e(\theta), y^e(\theta))]$

Definition 2 says that a a mechanism is first-best for for the funder if his payoff under the mechanism is equal to the maximum total surplus. Note that any payoff to the funder greater than $\max_{x(\theta),y(\theta)} E_\theta[\mathcal{S}(\theta, x(\theta), y(\theta))]$ would require that the researcher's ex-ante expected payoff is less than zero. Such a mechanism would violate voluntary participation.

With perfect information, the principal knows the true θ and all relevant research input is contractible. Note, however, there are two approaches one could take to abstract from the MH problem. The first approach would be to assume that both x and y are contractible. The second approach, which I adopt in the interest of keeping things as simple as possible, is to assume that the researcher faces a finite upper bound on effort, $\overline{y} < \infty$, and that effort is supplied at zero cost, $c = 0$. Under this specification, it is WLOG that $y = \overline{y}$. The qualitative conclusions do not change, however, under the contractible-y approach.

It is easy to see that the principal can achieve the first-best outcome using any combination of a grant or prize under perfect information. First, the funder calculates the socially efficient level of investment, $x^e \equiv \arg\max_x\{\theta\alpha(x, \overline{y})(W+\pi) - x\}$. He then requires the researcher to invest x^e to be eligible for the grant and/or prize. Finally, he offers a grant/prize combination that leaves the researcher indifferent between investing x^e or not:

$$g + \alpha(x^e, \overline{y})V = x - \alpha(x^e, \overline{y})\pi$$

When all relevant research input is contractible and there is no uncertainty over the researcher's ability, one can simply think of the expected value of a prize as a a transfer that the researcher receives with certainty. The socially efficient investment level is achieved and the funder attains the first-best. The next section studies the AS problem in isolation, I then analyze the MH problem in isolation, before combining the AS and MH problems.

4 Adverse Selection

This section studies the AS problem in isolation. To abstract from MH, assume that the researcher supplies effort at zero cost ($c = 0$); and that there is a finite upper bound on effort ($\overline{y} < \infty$). Then, WLOG fix the researcher's effort level at $\overline{y}$. Let $h(\theta) \equiv \frac{1-F(\theta)}{f(\theta)}$ denote the inverse hazard rate. As is standard in the literature, assume $h'(\theta) < 0$, and $f(\theta) > 0$ for all θ.

If $\pi = 0$, the funder could achieve the first-best through the use of a grant only (Bolton and Dewatripont, 2005, pp. 231).[20] To achieve this outcome, the funder chooses the efficient investment/effort schedules, and then reimburses the agent with a grant equal to the total cost incurred by the researcher. A researcher of any type is indifferent between following the funder's recommendation or not. However, when $\pi > 0$, this contract violates IC. As the efficient investment/effort levels are increasing in type, the researcher would always report being the highest type in order to maximize the probability of innovating and receiving π. Before proceeding, it is useful to re-write the maximand in (1) to reflect the fact that the funder's payoff is equal to the total surplus generated, less the rent that accrues to the researcher. The funder's problem may be reformulated:

$$\max_{\{x(\theta),V(\theta),U(\theta)\}} \int_{\Theta} [\theta\alpha(x(\theta),\overline{y})(W+\pi) - x(\theta) - U(\theta)]\, dF(\theta) \tag{4}$$

Subject to IR and IC. Using standard techniques, the proof of Proposition 1 shows that the funder's problem may be reformulated again as:

$$\max_{\{x(\theta),V(\theta)\}} \int_{\underline{\theta}}^{\overline{\theta}} \Big(\theta\alpha(x(\theta),\overline{y})[W+\pi] - x(\theta) - h(\theta)\alpha(x(\theta),\overline{y})(\pi + V(\theta))\Big)\, dF(\theta) \tag{5}$$

[20] This point is also made in Lewis and Sappington (2000b)

Subject to the following IC condition:

$$\frac{\partial}{\partial \theta}\left\{\alpha(x(\theta),\overline{y})[V(\theta)+\pi]\right\} \geq 0$$

It is clear from equation (5) that the funder's payoff is strictly decreasing in the size of the prize. Indeed, as the first result demonstrates, the optimal funding mechanism in the model with pure AS uses only a grant. The next proposition refers to $\underline{\theta}^m$; this is given by

$$\underline{\theta}^m = \max\{\underline{\theta}, \theta | \theta\alpha_1(0,\overline{y})(W+\pi) - 1 - h(\theta)\alpha_1(0,\overline{y})\pi = 0\}$$

Note that condition *T2* ensures $\underline{\theta}^m < \overline{\theta}$.

Proposition 1. *In the model with pure AS:*

(i) The optimal means of funding uses a grant only ($V^*(\theta) = 0$ for all θ).

(ii) For $\theta < \underline{\theta}^m$: $g^*(\theta) = x^*(\theta) = 0$; and for $\theta > \underline{\theta}^m$: $x^*(\theta) > 0$ and $g^*(\theta) > 0$, where $x^*(\theta)$ satisfies:

$$\theta\alpha_1(x^*(\theta),\overline{y})(W+\pi) - 1 - h(\theta)\alpha_1(x^*(\theta),\overline{y})\pi = 0 \tag{6}$$

(iii) For $\theta > \underline{\theta}^m$: the investment and grant schedules are strictly increasing with $g^*(\cdot) < x^*(\cdot)$ and $g^{*\prime}(\theta) < x^{*\prime}(\theta)$.

(iv) For $\theta < \overline{\theta}$ the investment level is strictly below the efficient level, so that $x^*(\theta) < x^e(\theta)$; but there is "efficiency at the top": $x^*(\overline{\theta}) = x^e(\overline{\theta})$.

To understand why no prize is offered, consider a simple two-type version of the model: $\Theta = \{\underline{\theta}, \overline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$. Since y is assumed fixed at $\overline{y}$, for clarity in the discussion that follows, I suppress this notationally. The funder's payoff is decreasing in the rent captured by the researcher; so, optimality dictates that the low type captures zero rent. But to ensure IC, the high type must be permitted to capture at least as much rent as she could by imitating the low type. If $\underline{z}$ is the reward for innovation received by the low type, then by under reporting, the high type can capture: $(\overline{\theta} - \underline{\theta})\alpha(x(\underline{\theta}))\underline{z} > 0$; and hence, the rent that accrues to the high type is proportional to the low type's reward for innovation. A prize therefore creates greater information rent for the higher type than does a grant. As these rents are costly to the funder, the optimal means of funding uses only a grant. Part (*ii*) of Proposition 1 characterizes the optimal investment schedule chosen by the funder. To provide some intuition for the optimal schedule (at an interior solution), re-arrange equation (6) to obtain:

$$\theta\alpha'(x^*(\theta))W = (1 - \theta\alpha'(x^*(\theta))\pi) + h(\theta)\alpha'(x^*(\theta))\pi$$

When an researcher of type θ invests x, the expected benefit to the funder is $\theta\alpha(x)W$. Hence, the left-hand side of the above equation is the marginal benefit of investment to the funder. The total cost incurred by the funder, when the type θ researcher invests x, may be broken up into two terms. First, the loss incurred by the researcher is $x - \theta\alpha(x)\pi > 0$. The funder must compensate the researcher to offset this loss and ensure participation. The first term on the right-hand side of the equation above, $1 - \theta\alpha'(x^*(\theta))\pi$, is thus the marginal cost of maintaining the individual rationality constraint. The term $h(\theta)\alpha(x)\pi$ may be thought of as the marginal cost to the funder of maintaining the incentive compatibility constraint. To understand this, first note that IC requires $U'(\theta) = \alpha(x)\pi$. So, the marginal effect of an increase in x on the slope of $U(\theta)$ is $\alpha'(x)\pi$. However, when the slope of $U(\cdot)$ increases at some particular value of θ, it must increase for all types just above θ as well. We can interpret $h(\theta)$ as the probability the researcher is of a type greater than θ, conditional on her type being in a neighborhood around θ; then $h(\theta)\alpha'(x)\pi$ gives the conditional-probability-weighted marginal change in the slope of $U(\theta)$ (w.r.t. x).

Parts (*iii*) and (*iv*) of Proposition 1 establish that the funder only partially reimburses the researcher for investment costs, and that the second-best investment path lies below the efficient path of investment. Both of these facts contrast results in Bolton and Dewatripont (2005) and Lewis and Sappington (2000b). Driving the difference is the presence, in my model, of the researcher's profit incentive, $\pi > 0$. Since the funder is unable to extract this profit from the researcher, this leaves information rents for any type above $\underline{\theta}$. The availability of information rents means that the funder cannot offer full reimbursement of investment and simultaneously sort over types; doing so would always create the incentive for the agent to invest as if she were the highest type – the type receiving the largest investment recommendation – in order to maximize the probability of innovating and receiving π. The optimal grant offers only partial reimbursement of investment, which may be interpreted as a matching grant. The matching feature requires the researcher to bear some of the cost of the project; this cost, $x(\theta) - g(\theta)$, is increasing in her reported type. As a result, only a researcher of high ability, who is sufficiently likely to innovate, is willing to report being a high type. Maurer and Scotchmer also make the point that a matching grant can be an effective screening device in the presence of AS; Proposition 1, reveals that this may indeed be the optimal screening device.

5 Moral Hazard

This section studies MH in isolation; so let $c > 0$ be the researcher's effort cost. But to abstract from AS, suppose F places unit mass on some $\tilde{\theta} \in \Theta$. In what follows, it is convenient to to have a measure of the strength/severity of the MH problem; the magnitude of the ratio, $\frac{c}{\pi}$, gives one such measure. Ceteris paribus; the larger this ratio, the weaker is the incentive for the researcher to expend effort, and hence, the stronger/more severe is the MH problem. The funder's problem with only MH can be written:

$$\max_{\{x,y,g,V\}} \left\{ \tilde{\theta}\alpha(x,y)\big[W - V\big] - g \right\}$$

subject to the non-negativity constraints and

$$\tilde{\theta}\alpha(x,y)\big[\pi + V\big] - x - cy + g \geq 0 \tag{7}$$

$$y = \arg \max_{y' \in [0,\overline{y}]} \{\tilde{\theta}\alpha(x,y')(V+\pi) - cy'\} \tag{8}$$

Equation (7) gives the IR constraint, and equation (8) gives the IC constraint, which requires that the researcher's effort be chosen optimally. Strict concavity of the researcher's effort problem ensures that, when y is interior, (8) can be replaced by the first-order condition $\tilde{\theta}\alpha_2(x,y)(V+\pi) - c = 0$. The first result in this section shows that with pure MH, prizes are generally preferred by the funder.

Proposition 2.
In the model with pure MH, there exists an optimal funding scheme in which only a prize is offered (i.e. $g^* = 0$).

The intuition for Proposition 2 is in line with standard theory on MH; by only rewarding success, a prize creates a greater incentive for unobservable effort than does a grant. Indeed, examining the researcher's effort problem in equation (8) it is clear that her effort choice is completely independent of the size of the grant. Even so, a grant can elicit higher levels of unobservable effort. The logic is as follows: a grant may be used to in induce higher levels of contractible investment; higher levels of investment increase the marginal returns to effort; as there is some profit incentive for R&D, the researcher exerts greater effort.

Not only is a grant relevant in my model with pure MH; the final result in this section shows that, under some conditions, a grant may actually be an optimal means of funding. The hypothesis of the proposition is that the ratio, $\frac{c}{\pi}$, is sufficiently small (the precise bound is given in the proposition). Recall that $\frac{c}{\pi}$ gives one measure of the strength or severity of the MH problem. As the agent's effort cost (profit incentive) increases (decreases), the incentive to exert effort diminishes, and the principal faces a more severe MH problem. Thus, the condition given in the proposition can be interpreted as a statement restricting the severity of the MH problem.

Proposition 3.

Suppose the researcher's effort supply is limited ($\overline{y} < \infty$) and $\frac{c}{\pi} < \tilde{\theta}\alpha_2(x^e, \overline{y})$, then there exists an optimal means of funding that uses only a grant ($V^* = 0$). Moreover, the funder may achieve the first-best.

6 Combined Adverse Selection and Moral Hazard

This section studies the model in the presence of both AS and MH; so, suppose the researcher's ability, θ, is unknown to the funder and her effort cost is non-zero: $c > 0$. As shown in Section 4, grants are a more efficient means of dealing with the AS problem, but as shown in Section 5 prizes tend to be a more effective means of dealing with MH. In consequence, when both informational problems are relevant, the optimal means of funding typically involves some combination of a grant and prize. The first result shows, however, if the MH problem is sufficiently weak, then the optimal means of funding uses only a grant. In what follows, let $x^{AS}(\theta)$ and $g^{AS}(\theta)$ respectively denote the optimal investment and grant schedules characterized in Proposition 1 when the funder faces a pure AS problem.

Proposition 4.
Suppose the researcher's effort supply is limited ($\overline{y} < \infty$), and the MH problem is sufficiently weak: $\frac{c}{\pi} < \underline{\theta}\alpha_2(x^{AS}(\underline{\theta}), \overline{y})$. Then

(i) The optimal means of funding uses only a grant ($V^* = 0$).

(ii) The optimal investment and grant schedules are, respectively $x^*(\cdot) = x^{AS}(\cdot)$ and $g^*(\cdot) = g^{AS}(\cdot) + c\overline{y}$.

(iii) For all $\theta \in \Theta$: $g^*(\theta) < x^*(\theta)$ and $g^{*'}(\theta) < x^{*'}(\theta)$.

The key condition for Proposition 4 is the bound placed on the ratio $\frac{c}{\pi}$; this condition places a restriction on the strength of the MH problem. Specifically, when the MH problem is sufficiently weak then – even without a prize – the investment level, $x^{AS}(\theta)$, is sufficient to induce a researcher of any type to exert maximal effort. Hence, the MH problem can be completely overcome by providing incentives solely for investment. The comparative advantage of the prize – providing strong incentives for unobservable effort – is mitigated. To limit the informational rent that accrues to the researcher (due to the AS problem), the funder uses only a grant.

One of the benefits of a grant is that it provides a resource-constrained researcher with a source of up-front funding. Yet an appropriately designed pull program *could* overcome this issue. A prize, for example, should have the ability to attract financiers to share in the rewards. Indeed, this is exactly the logic behind the U.S. Department of Labor's "Pay for Success" model.[21] The researcher in my model does not face an explicit constraint on capital; nevertheless, Proposition 4 helps to shed light on the issue. Taken together, the hypotheses of Proposition 4 and condition *P1* describe a setting in which the researcher: (1) has a strong incentive to devote her time and energy to R&D, but (2) is unwilling to bear the cost of acquiring the necessary capital. Such conditions may well arise because the researcher faces strong frictions in the financial sector. An appropriately sized prize could overcome these frictions: the researcher would be willing to incur the high cost of capital if the potential benefit were large enough. Nevertheless, Proposition 4 reveals that – in the presence of AS – a grant is a more efficient way to deal with this type of friction.

Given the efficiency of prizes in dealing with pure MH (see Section 5), one might expect that in an environment with both AS and MH, prizes would be somehow more attractive than in a setting with only AS. Contrary to

[21] According to the Department of Labor:

> Under the Pay for Success model, a government agency commits funds to pay for a specific outcome that is achieved within a given timeframe. The financial capital to cover the operating costs of achieving the outcome is provided by independent investors. In return for accepting the risks of funding the project, the investors may expect a return on their investment if the project is successful; however, payment of the committed funds by the government agency is contingent on the validated achievement of results. `http://www.doleta.gov/workforce_innovation/success.cfm`

this intuition, it happens that prizes – in some sense – actually become *less* attractive than grants, when a (weak) MH problem is added on top of an AS problem. To convey the idea, suppose the hypotheses of Proposition 4 are satisfied, and suppose $\overline{y} = 1$. Then the total effort cost, when the agent chooses maximal effort, is $c > 0$. As compared to the pure AS environment, the funder's equilibrium payoff decreases by exactly c, while the researcher captures exactly as much rent. This follows since, as compared to the pure AS environment, the grant schedule offered to the researcher increases by exactly c, while the path of investment/effort remain unchanged. But suppose the funder – recognizing the MH problem – incorrectly concludes that he should use only a prize. As compared to the pure AS environment, the prize offered to the lowest type must increase by just enough to offset the additional effort cost, in order to maintain IR. But increasing the size of the prize offered to this low type generates additional information rents for higher types. To maintain IC, the funder must allow these higher types to capture even more rent than in the pure AS environment. This means the expected value of the prizes offered to higher types must increase by more than c, and the expected payoff to the funder thus decreases by more than c. Indeed, higher values of c exacerbate the AS problem and may – in some cases – *decrease* the relative attractiveness of prizes.

The next result formalizes this idea. To keep the analysis as simple as possible, suppose effort is binary: $y \in \{0, 1\}$. If the researcher does not exert effort, $y = 0$, the project fails with certainty. If the researcher exerts effort, $y = 1$, then she incurs cost, c, and the probability of success is $\theta\alpha(x)$. If investment is x and the reward for successfully innovating is z then the researcher exerts effort only if: $\theta\alpha(x)z - c \geq 0$.

Let $\phi^*(c)$ denote the funder's payoff under the optimal funding scheme; and let $\phi^p(c)$ denote the funder's maximal payoff if she uses only a prize. In this binary-effort specification, the bound provided in Proposition 4 on $\frac{c}{\pi}$ reduces to $\frac{c}{\pi} \leq \underline{\theta}\alpha(x^{AS}(\underline{\theta}))$. So, if this condition is satisfied then the optimal means of funding uses only a grant and $\phi^*(\theta)$ is the funder's payoff when he uses a pure grant. Finally, define the function $\tilde{h}(\theta)$:

$$\tilde{h}(\theta) \equiv \frac{\int_{\theta}^{\overline{\theta}} t f(t)\, dt}{\theta^2 f(\theta))} \tag{9}$$

For the remainder of this section, I assume $\tilde{h}$ is strictly decreasing; this assumption is qualitatively similar to the decreasing inverse hazard rate assumption. Both conditions rule out distributions with long, thin tails, and are satisfied for distributions such as the uniform, the triangular distribution, and many parameterizations of the beta distribution.

Proposition 5.
For all $c \geq 0$ such that $\frac{c}{\pi} \leq \underline{\theta}\alpha(x^{AS}(\underline{\theta}))$: the difference between the funder's grant and prize payoffs, $\phi^*(c) - \phi^p(c)$, is strictly increasing in c.

The final result characterizes the optimal means of funding, under the binary-effort specification, when the MH problem is relatively severe. In this case, a prize is essential to induce effort; however, a grant still plays an important role in the funding scheme, as a way to limit the information rent that accrues to the researcher.

Proposition 6.
Suppose the moral hazard problem is sufficiently strong: $\frac{c}{\pi} > \overline{\theta}\alpha(x^e(\overline{\theta}))$. Then, in the binary-effort specification of the model with both AS and MH, there exists $\theta^* \in [\underline{\theta}, \overline{\theta})$ such that, for all $\theta < \theta^*$: $x^*(\theta) = g^*(\theta) = V^*(\theta) = 0$. For all $\theta \in [\theta^m, \overline{\theta}]$:

(i) the optimal investment path is efficient: $x^*(\theta) = x^e(\theta)$, but the funder does not achieve the first best

(ii) the grant offers full reimbursement of investment: $g^*(\theta) = x^*(\theta)$

(iii) the prize schedule is strictly decreasing and is given by, $V^*(\theta) = \frac{c}{\theta^m \alpha(x^*(\theta))} - \pi$

Under the hypotheses of Proposition 6, the MH problem is severe, and a prize is necessary to elicit effort from a researcher of any type. Higher types require smaller prizes to elicit effort, and so the optimal prize schedule is decreasing in the researcher's reported type. But what purpose does the grant serve in this contract? To address this question, consider fixing all the parameters of the model, but let c vary in such a way that we move from the case with AS only ($c = 0$), to the case with weak MH/AS (Proposition 4), and then to the case given in Proposition 6. Then, relative to the case with weak MH/AS or AS only, the investment schedule with strong MH increases (for all types above θ^*). The reason is that a higher level of investment reduces the size of the required prize, which limits the information rents for the researcher. But this large investment increases the researcher's loss, absent the incentives provided by the funder. Since prizes generate large information rents, the funder uses a grant – rather than a prize – to offset the researcher's loss.

It is also worth discussing the funder's optimal choice of θ^* – the lowest type that receives a positive investment recommendation. It can be shown that under the investment/prize/grant schedules given in the proposition, the funder's instantaneous payoff is strictly positive at each $\theta \in \Theta$. So why might the funder choose $\theta^* > \underline{\theta}$? To provide the intuition, first note that the size of the prize offered a researcher of type θ^* is just large enough to make a researcher of that type indifferent between exerting effort or not. The lower is θ^*, the larger is the necessary prize necessary for this type. In turn, this increases the information rent for all types above θ^*. Hence, a decrease in θ^* has two competing effects on the funder's payoff: first, it increases the probability that the funder offers a non-zero contract, and earns non-zero surplus. Second, it increases the information rent for all types above θ^*. The optimal choice of θ^* balances these two forces.[22] Finally, I point out that the mechanism characterized in Proposition 6 is weakly implementable, in the sense that an researcher of any type is indifferent between reporting truthfully and not. If a high type underreports, she receives a larger prize, but also invests less, meaning she is less likely to succeed and receive the prize. The

[22]Note that the investment schedule characterized in Proposition 6 may be discontinuous at the point θ^*, as investment jumps from zero to $x^e(\theta^*) > 0$; this feature is driven by the binary-effort specification of the model.

optimal contract balances these two effects in such a way that the researcher is indifferent between truthful reporting and not.

7 Comparative Statics

Thus far, the analysis has focused on the relative performance of push and pull programs in various informational environments. In this section, I explore some of the more interesting comparative statics results, focusing on the pure AS and MH environments.

Adverse Selection

One interesting feature of the optimal contract under pure AS is the relationship between the optimal investment path and π. Intuitively, one might expect more profitable projects to receive larger investments; with perfect information, this is indeed the case. The first result shows, however, that in the presence of AS this need not be the case. That is, the investment path (at some particular value of θ) may either increase or decrease in π. Consistent with the perfect information environment, however, an increase in π increases the (ex-ante expected) payoff to the principal.

Proposition 7. *For the optimal funding scheme characterized in Proposition 1:*

(i) At any fixed $\theta \in \Theta$: $\frac{\partial x^*(\theta)}{\partial \pi} > 0$ if and only if $\theta > h(\theta)$.

(ii) The funder's ex-ante expected payoff is strictly increasing in π.

Recall from the discussion that followed Proposition 1, that the marginal cost to the funder of incentivizing a type θ agent to invest x may be broken up into two terms. First, the term $1 - \theta\alpha'(x)\pi$ can be interpreted as the marginal cost to the funder of maintaining IR. Clearly, this term is decreasing in π. At the same time, the marginal cost of maintaining IC is $h(\theta)\alpha'(x)\pi$, which is clearly increasing in π. So, an increase in π has two competing effects on the funder's marginal costs; the net effect on the optimal investment path depends on the balance of these two forces. Note that since $h(\overline{\theta}) = 0$ the IR effect always dominates the IC effect for types close to $\overline{\theta}$. Proposition 7 also reveals that an increase in π unambiguously increases the funder's expected payoff. An increase in π has two effects on (expected) total cost: it reduces the expected total cost of maintaining IR, but increases the expected total cost of maintaining IC. On average, the IR effect on total cost outweighs the IC effect on total cost, and the funder's expected payoff increases with π.

Finally, one may wonder whether more profitable projects should receive larger or smaller grants. Unfortunately, the effect of a change in π on the grant path is unclear due to two sources of ambiguity. A grant serves two distinct purposes: it is necessary to offset losses incurred by the researcher (to maintain IR), and it is used to reward higher types with information rents (to maintain IC). The first source of ambiguity arises from the impact of a change in π on the size of the grant necessary to maintain IR. There is a direct effect: an increase in π means greater available profit to the researcher, and hence a smaller grant required to maintain IR. But there is also an indirect effect: an increase in π may lead the funder to either increase or decrease the investment level at θ. Ceteris paribus, an increase (decrease) in the investment level means a larger (smaller) grant is necessary to maintain IR.[23] The second source of ambiguity arises from the impact of a change in π on the grant size required to maintain IC. There is a direct effect: an increase in π increases the reward for innovation, which increases the amount of rent the type-θ researcher can can capture by imitating a lower type. Ceteris paribus, this means that a larger grant is necessary to maintain IC. But there is also an indirect effect, since the investment level – in some neighborhood about θ – may increase or decrease. All else equal, an increase (decrease) in the investment level for types in some neighborhood below θ increases (decreases) the available information rent to the type θ researcher, and hence increases (decreases) the size of the grant necessary to maintain IC. The net effect of a change in π on the size of the

[23]This observation follows from our assumption that, absent any other incentive, the researcher's profit is strictly decreasing in her investment

grant (at θ) depends on the balance of these four forces and is, in general, ambiguous.[24]

Moral Hazard

There are two relationships I wish to highlight in the model with MH. First, in contrast to the AS model, with pure MH the optimal investment level (and effort level) is always increasing in π. Intuitively, an increase in π leads the researcher to exert greater effort, ceteris paribus. This relaxation of the MH problem makes each unit of investment more productive from the principal's perspective. Moreover, an increase in π increases the total surplus that the funder can appropriate. Both of these effects lead the principal to require a greater investment from the researcher, ceteris paribus. While an increase in π means the funder may offer a smaller prize, the total reward for innovation, $V^* + \pi$, is always increasing in π. Consistent with the model under AS, an increase in π increases the funder's expected payoff under the optimal contract. I also investigate the effect of the researcher's ability, $\tilde{\theta}$, on the optimal contract. Consistent with the model with pure AS, a more able researcher invests more (and exerts greater effort). While the prize may decrease in response to an increase in $\tilde{\theta}$, the ability-weighted reward for innovation, $\tilde{\theta}(V^* + \pi)$, always increases. The intuition is straightforward, and consistent with the model under AS: an increase in the researcher's ability increases the marginal returns to both investment and effort.

In what follows let ϕ^* denote the funder's payoff under the optimal MH contract. For ease of exposition, take $\overline{y}$ to be arbitrarily large and $\tilde{\theta}\alpha_2(x, 0)\pi - c > 0$ for all $x > 0$. Under these conditions, for any $x > 0$ and $V \geq 0$, the researcher's effort choice, y^*, satisfies the first-order condition:

$$\tilde{\theta}\alpha_2(x, y^*)(V + \pi) - c = 0$$

Proposition 8. *For the optimal funding scheme under pure MH:*

[24] Indeed, it is straightforward to construct examples in which the grant path increases or decreases in π for all θ.

(i) $\frac{\partial x^*}{\partial \pi} \geq 0$, $\frac{\partial y^*}{\partial \pi} \geq 0$, $\frac{\partial}{\partial \pi}(V^* + \pi) \geq 0$, *and* $\frac{\partial \phi^*}{\partial \pi} > 0$

(ii) $\frac{\partial x^*}{\partial \tilde{\theta}} \geq 0$, $\frac{\partial y^*}{\partial \tilde{\theta}} \geq 0$ *and* $\frac{\partial}{\partial \tilde{\theta}}\left[\tilde{\theta}(V^* + \pi)\right] \geq 0$, *and* $\frac{\partial \phi^*}{\partial \tilde{\theta}} > 0$

8 Discussion and Conclusion

The results herein are relevant for understanding the means by which funders encourage R&D in many areas. One particular setting involves the development of certain pharmaceuticals. Specifically, pharmaceuticals used to treat certain "neglected tropical diseases" (NTDs), which are health conditions that disproportionately affect the developing world. The Center for Disease Control estimates that more than one billion people are infected with one or more of these conditions.[25] Due to market distortions, the profitability of treatments for NTDs is low; hence, treatments and vaccines are slow to develop: NTDs carry over 10% of the global disease burden, yet of the 1,393 new drugs brought to market from 1974-1999 only 16 were for NTDs (Yamey, 2002).[26] The social value of new treatments, however, is immense.[27]

The Bill and Melinda Gates Foundation (GF) encourages R&D activity for new pharmaceuticals, and has devoted significant resources to both push and pull programs. In July, 2013, for instance, GF awarded a $160 million dollar grant to the Medicines for Malaria Venture – a non-profit that seeks to develop treatments for Malaria. GF awards the majority of its grants to nonprofit research organizations, as they explicitly state: "The foundation awards the majority of its grants to U.S. 501(c)(3) organizations and other tax-exempt organizations...".[28] Yet, GF has also committed resources to the new Advanced Market Commitment program for pneumococcal vaccines (AMC). The AMC

[25] `http://www.cdc.gov/globalhealth/ntd/`

[26] In this market, underinvestment has resulted from a collective action problem, and a potential hold-up problem. See Glennerster et al. (2006) for a detailed discussion of these market failures.

[27] Gallup and Sachs (2000), for example, present evidence for a significant causal link between malaria and poverty; they estimate that the eradication of malaria, in a nation significantly affected by the disease, would increase economic growth by about 2.6%.

[28] `http://www.gatesfoundation.org/How-We-Work/General-Information/Grant-Opportunities`

is a pull mechanism that guarantees a viable market for certain pneumococcal vaccines; it does not distinguish between nonprofits and for-profits.

The preferred method of funding nonprofits vs. for-profits seems casually consistent with the predictions of my model. It is not difficult to imagine that GF faces an AS problem. The development of pharmaceuticals requires highly specialized knowledge; researchers may hold private information about the viability of certain projects, given their experience with, and portfolio of, related projects. It is also easy to imagine that GF faces a more severe MH problem in funding for-profits. Since for-profits likely have more profitable business ventures, the opportunity cost of devoting time and energy to NTD treatments is high, relative to the profit. Nonprofit research foundations – established with the specific goal of developing treatments for NTDs – likely face a lower opportunity-cost-to-benefit ratio.[29] Nonprofits may, however, face a higher cost of acquiring the necessary capital, as they largely rely on costly fundraising activities.[30] To translate these observations to the language of my model, it seems that for nonprofits: the ratio, $\frac{c}{\pi}$, is small, while the cost of capital is large, relative to π. It is precisely this environment in which my model predicts grants to be the preferred means of funding (see Proposition 4, and the discussion that follows). For-profit firms likely face less friction in capital markets, but $\frac{c}{\pi}$ may be large; it is precisely such a setting in which my model predicts the necessity of a pull incentive.

To conclude, this paper has studied the optimal means of funding in a setting with AS and MH, allowing for the presence of an observable input. In a pure AS environment, a matching grant emerges as the optimal means of funding. In a pure MH environment a prize is generally the optimal means of funding. Still, I highlight the relevance of grants in such settings: the grant can be used to encourage observable monetary investment, which increases the marginal returns to effort. In the presence of both AS and MH, a push program may emerge as the optimal means of funding. The environment in

[29]For these nonprofits, π may be largely non monetary.

[30]The Medicines for Malaria Venture, for example, receives all of its funding from public and private donations/grants. `http://www.mmv.org/invest-in-us/mmv-funding-and-expenditure`

which the push program is optimal can be interpreted as one in which the researcher has a strong incentive to devote her time to a project, but faces strong frictions in the financial sector.

References

Arrow, K. J. (1970). Essays in the theory of risk-bearing.

Bolton, P. and Dewatripont, M. (2005). *Contract theory*. MIT press.

Chari, V. V., Golosov, M., and Tsyvinski, A. (2012). Prizes and patents: Using market signals to provide incentives for innovations. *Journal of Economic Theory*, 147(2):781–801.

Chen, Y. (2013). *Observable actions in generalized moral hazard problems*. PhD thesis, Indiana University.

Choi, J. P. (1992). Cooperative R&D with moral hazard. *Economics Letters*, 39(4):485–491.

De Laat, E. A. (1997). Patents or prizes: Monopolistic r&d and asymmetric information. *International Journal of Industrial Organization*, 15(3):369–390.

Fu, Q., Lu, J., and Lu, Y. (2012). Incentivizing r&d: Prize or subsidies? *International Journal of Industrial Organization*, 30(1):67–79.

Gallup, J. and Sachs, J. (2000). The economic burden of malaria. *The American journal of tropical medicine and hygiene*, 64(1-2 Suppl):85–96.

Glennerster, R., Kremer, M., and Williams, H. (2006). Creating markets for vaccines. *innovations*, 1(1):67–79.

Gottlieb, D. and Moreira, H. (2014). Simultaneous adverse selection and moral hazard. Technical report, Mimeo.

Gottlieb, D. and Moreira, H. (2015). Simple contracts with adverse selection and moral hazard. *The Wharton School Research Paper*, (78).

Hall, B. H., Mairesse, J., and Mohnen, P. (2009). Measuring the returns to R&D. National Bureau of Economic Research w15622.

Hemel, D. J. and Ouellette, L. L. (2013). Beyond the patents-prizes debate. *Texas Law Review*, 92(2):303.

Hölmstrom, B. (1979). Moral hazard and observability. *The Bell journal of economics*, pages 74–91.

Holmstrom, B. and Milgrom, P. (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics, & Organization*, pages 24–52.

Hopenhayn, H., Llobet, G., and Mitchell, M. (2006). Rewarding sequential innovators: Prizes, patents, and buyouts. *Journal of Political Economy*, 114(6):1041–1068.

Innes, R. D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory*, 52(1):45–67.

Jones, C. I. and Williams, J. C. (1998). Measuring the Social Return to R&D. *The Quarterly Journal of Economics*, 113(4):1119–1135.

Kremer, M. (2002). Pharmaceuticals and the developing world. *Journal of Economic Perspectives*, pages 67–90.

Laffont, J.-J. and Tirole, J. (1993). *A theory of Incentives in Procurement and Regulation*. Cambridge, Mass: MIT Press.

Lazear, E. P. (1986). Salaries and piece rates. *Journal of business*, pages 405–431.

Lazear, E. P. (2000). The power of incentives. *American Economic Review*, pages 410–414.

Lewis, T. R. and Sappington, D. E. (2000a). Contracting With WealthConstrained Agents. *International Economic Review*, 41(3):743–767.

Lewis, T. R. and Sappington, D. E. (2000b). Motivating wealth-constrained actors. *American Economic Review*, pages 944–960.

Lewis, T. R. and Sappington, D. E. (2001). Optimal contracting with private knowledge of wealth and ability. *The Review of Economic Studies*, 68(1):21–44.

Maurer, S. M. and Scotchmer, S. (2003). Procuring knowledge. Technical report, National Bureau of Economic Research.

Reider, C. R. (2000). The orphan drug act: provisions and considerations. *Drug information journal*, 34(1):295–300.

Scotchmer, S. (1999). On the optimality of the patent renewal system. *The RAND Journal of Economics*, pages 181–196.

Spence, M. and Zeckhauser, R. (1971). Insurance, information, and individual action. *American Economic Review*, pages 380–387.

Stiglitz, J. E. (1974). Incentives and risk sharing in sharecropping. *The Review of Economic Studies*, pages 219–255.

Weyl, E. G. and Tirole, J. (2012). Market Power Screens Willingness-to-Pay. *Quarterly Journal of Economics*, 127(4):1971–2000.

Wright, B.-D. (1983). The economics of invention incentives: Patents, prizes, and research contracts. *The American Economic Review*.

Yamey, G. (2002). The world's most neglected diseases: Ignored by the pharmaceutical industry and by public-private partnerships. *BMJ: British Medical Journal*, 325(7357):176.

Appendix A: Proofs

Proof of Proposition 1. I first show that the problem given in equation (4) can be expressed as in equation (5). That the payoff to a type θ researcher that reports $\hat{\theta}$ is

$$U(\hat{\theta}|\theta) = \theta\alpha(x(\hat{\theta}), \overline{y})(V(\theta) + \pi) - x(\hat{\theta}) + g(\hat{\theta})$$

The incentive compatibility constraint (IC) requires $U(\theta|\theta) \geq U(\hat{\theta}|\theta)$ for all $\theta, \hat{\theta} \in \Theta$. The IC constraint is satisfied if and only if $U_1(\theta|\theta) = 0$ and $U_{12}(\theta, \theta) \geq 0$ for all $\theta \in \Theta$ (A proof found in Laffont and Tirole, 1993, pp. 64 and 121 may easily be adapted to the present context to demonstrate this result.) Using the envelope theorem, $U_1(\theta|\theta) = 0$ implies $U'(\theta) = \alpha(x(\theta), \overline{y})(\pi + V(\theta))$. Then, $U_{12}(\theta, \hat{\theta}) \geq 0$ if and only if for all θ:

$$\alpha_1(x(\theta), \overline{y})x'(\theta)(V(\theta) + \pi) - \alpha(x(\theta), \overline{y})V'(\theta) \geq 0$$

Condition *P1* ensures that, absent any other incentives, the researcher invests nothing, and earns zero profit. Thus, the individual rationality (IR) constraint requires $U(\theta) \geq 0$ for all θ. Since $U'(\theta) \geq 0$, the IR constraint is satisfied so long as $U(\underline{\theta}) \geq 0$. But from equation (4) it is clear that the funder's payoff is decreasing in $U(\cdot)$, and hence the funder optimally sets $U(\underline{\theta}) = 0$. So, the IR and two IC constraints may, respectively, be summarized:

(IR) $U(\underline{\theta}) = 0$

(IC1) $U'(\theta) = \alpha(x(\theta), \overline{y})(\pi + V(\theta))$

(IC2) $\alpha_1(x(\theta), \overline{y})x'(\theta)(V(\theta) + \pi) - \alpha(x(\theta), \overline{y})V'(\theta) \geq 0$

(IR) and (IC1) together imply:

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \alpha(x(\theta), \overline{y})(\pi + V(\theta))\, d\theta$$

Integrating by parts, it follows:

$$\int_{\underline{\theta}}^{\overline{\theta}} U(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} h(\theta)\alpha(x(\theta), \overline{y})(\pi + V(\theta))\, dF(\theta)$$

Plugging this expression into equation (4), we obtain the problem formulated in (5). Examining the funder's payoff in (5), it is clear that the funder's payoff is strictly decreasing in $V(\cdot)$, and hence she optimally sets $V(\theta) = 0$ for all θ, so long as this does not violate (IC2). Note that when $V(\theta) = 0$ for all θ then $(IC2)$ is satisfied if and only if $x'(\theta) \geq 0$. Ignore this constraint; I will show that it is never binding.

Differentiating the maximand in (5) with respect to $x(\cdot)$, we obtain the point wise first order condition given in equation (6):

$$\alpha_1(x^*(\theta), \overline{y})[\theta(W + \pi) - h(\theta)\pi] - 1 \leq 0$$

Holding with equality when $x^*(\theta) > 0$. Note that for $\theta < \underline{\theta}^m$: $\alpha_1(x, \overline{y})[\theta(W + \pi) - h(\theta)\pi] - 1 < 0$ for any $x > 0$ and hence, $x^*(\theta) = 0$ for $\theta \leq \underline{\theta}^m$. Since $h(\cdot)$ is strictly decreasing, $\theta > \underline{\theta}^m$ implies $\alpha_1(0, \overline{y})[\theta(W+\pi) - h(\theta)\pi] - 1 > 0$, which means $x^*(\theta) > 0$ for $\theta > \underline{\theta}^m$. Next, note that $\theta > \underline{\theta}^m$ implies $\theta(W+\pi) - h(\theta)\pi > 0$. It is straightforward to show that that this means the maximand in (5) is strictly concave in $x(\cdot)$ for all $\theta > \theta^m$. Differentiating (6) with respect to θ, it is straightforward to show $h'(\theta) < 0$ and $\theta(W + \pi) - h(\theta)\pi > 0 \implies x^{*'}(\theta) > 0$, ensuring that (IC2) is satisfied. Hence, for $\theta > \underline{\theta}^m$: (6) characterizes the optimal level of investment. This establishes (i) and (ii). Next, see that $U_1(\theta|\theta) = 0$ implies:

$$x^{*'}(\theta)[\theta\alpha_1(x^*(\theta), \overline{y})\pi - 1] + g^{*'}(\theta) = 0 \tag{10}$$

$\theta > \theta^m \implies x^{*'}(\theta) > 0$; this fact along with Condition *P1* means that equation (10) implies $g^{*'}(\theta) > 0$. To complete the proof of part (iii) note that $U(\underline{\theta}^m) = 0$ and this implies $x(\underline{\theta}^m) - g(\underline{\theta}^m) = \underline{\theta}^m \alpha(x(\underline{\theta}^m), \overline{y})\pi \geq 0$, and hence, $g(\underline{\theta}^m) \leq x(\underline{\theta}^m)$. Finally, (10) gives: $x^{*'}(\cdot) - g^{*'}(\cdot) = x^{*'}(\cdot)\theta\alpha_1(\cdot)\pi > 0$, which means $g^*(\theta) < x^*(\theta)$ for all $\theta > \underline{\theta}^m$, establishing (iii). Now, note that the efficient investment schedule, $x^e(\theta)$, satisfies:

$$\theta\alpha_1(x^e(\theta),\overline{y})(W+\pi)-1=0$$

Since $h(\theta)>0$ for all $\theta<\overline{\theta}$, equation (6) implies $x^*(\theta)<x^e(\theta)$ for all $\theta<\overline{\theta}$, and since $h(\overline{\theta})=0$ we have $x^*(\overline{\theta})=x^e(\overline{\theta})$, establishing (iv). □

Proof of Proposition 2. To establish the proposition it suffices to show that for any feasible contract in which a grant is offered, there exists another feasible contract, which uses only a prize, and gives at least as high a payoff to the funder. So, let $\mathcal{C}=\{x,y,V,g\}$ denote an arbitrary feasible contract and suppose $g>0$. Since $g>0$, the claim would follow trivially if $\alpha(x,y)=0$, or if $W\leq V$. Hence, WLOG, assume $\alpha(x,y)>0$, and $W>V$. Let $\tilde{V}=V+\frac{g}{\tilde{\theta}\alpha(x,y)}$, and consider the new contract: $\tilde{\mathcal{C}}=\{x,\tilde{y},\tilde{V},0\}$ where $\tilde{y}$ is the researcher's optimal effort choice when investment is x and the prize is $\tilde{V}$. Let $U(\mathcal{C})$ and $U(\tilde{\mathcal{C}})$ respectively denote the researcher's payoff under the contracts $\mathcal{C}$ and $\tilde{\mathcal{C}}$. I first show that $\tilde{\mathcal{C}}$ is feasible. By construction, $\tilde{y}$ is the optimal effort choice for the researcher; and hence, feasibility of $\tilde{\mathcal{C}}$ just requires showing the IR constraint is satisfied:

$$\begin{aligned} U(\tilde{\mathcal{C}}) &= \tilde{\theta}\alpha(x,\tilde{y})(\pi+\tilde{V})-x-c\tilde{y} \\ &\geq \tilde{\theta}\alpha(x,y)(\pi+\tilde{V})-x-cy \\ &= U(\mathcal{C}) \\ &\geq 0 \end{aligned}$$

The inequality follows from the definition of $\tilde{y}$, the second equality follows from the definition of $\tilde{V}$, and the final equality is true by the feasibility of $\mathcal{C}$. Thus, $\tilde{\mathcal{C}}$ is feasible. It is straightforward to show that the researcher's optimal effort choice is non decreasing in the reward for innovation, for fixed x (and is independent of the size of the grant), and hence $\tilde{y}\geq y$ (holding strictly if y is interior: $y\in(0,\overline{y})$). Let $\phi(\mathcal{C})$ and $\phi(\tilde{\mathcal{C}})$ denote the funder's payoff under $\mathcal{C}$ and $\tilde{\mathcal{C}}$, respectively. See that:

$$\begin{aligned}\phi(\tilde{\mathcal{C}}) &= \tilde{\theta}\alpha(x,\tilde{y})(W-\tilde{V}) \\ &= \tilde{\theta}\alpha(x,\tilde{y})(W-V)-g \\ &\geq \tilde{\theta}\alpha(x,y)(W-V)-g \\ &= \phi(\mathcal{C})\end{aligned}$$

The second equality follows from the definition of $\tilde{V}$, and the inequality holds since $\tilde{y} \geq y$ and $W > V$. □

Proof of Proposition 3. The hypothesis of the proposition gives: $\tilde{\theta}\alpha_2(x^e, \overline{y})(W+\pi) - c > 0$; and this means $y^e = \overline{y}$. Then, x^e solves: $\tilde{\theta}\alpha_1(x^e, \overline{y})(W+\pi) - 1 = 0$. We will show that the funder can achieve the first-best without the use of a prize. Thus, we set $V = 0$. Let x^* be the optimal investment level chosen by the funder, and suppose that the researcher's effort constraint binds: $y(x^*, \pi) = \overline{y}$; we will then verify that this is the case. The funder solves:

$$\max_{x,g}\{\tilde{\theta}\alpha(x,\overline{y})W - g\} \text{ such that } \tilde{\theta}\alpha(x,\overline{y})\pi - x - c\overline{y} + g \geq 0$$

Note that the IR constraint implies $g > 0$; moreover, this constraint must bind. If not, the funder could decrease g slightly and increase her payoff. Then, using the constraint to eliminate g from the problem, and taking the first-order condition yields:

$$\tilde{\theta}\alpha_1(x^*, \overline{y})(W+\pi) - 1 = 0$$

So $x^* = x^e$; and hence, $y(x^*, \pi) = y(x^e, \pi) = \overline{y}$, which verifies that the researcher's effort constraint must bind. Thus, the funder can achieve the first-best through a grant only. □

Proof of Proposition 4. First, consider an auxiliary problem where both x and y are contractible. The auxiliary problem is a strict relaxation of the funder's problem, and so it provides an upper bound on his payoff. Under the conditions of the proposition, I will show that the funder may achieve this upper bound with the stated investment/grant schedules. Following nearly identical arguments as in the proof of proposition 1, when the principal chooses both x and y, he sets $V^* = 0$, and his problem can be expressed:

$$\max_{x(\cdot),y(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} [\theta\alpha(x(\theta), y(\theta))(W + \pi) - x(\theta) - cy(\theta) - h(\theta)\alpha(x(\theta), y(\theta))\pi]dF(\theta)$$

Subject to non negativity constraints, the effort constraint $y \leq \overline{y}$, and the IC condition $\alpha_1(\cdot)x'(\theta) + \alpha_2(\cdot)y'(\cdot) \geq 0$. For now, we ignore the IC condition; but note that if $y^*(\theta) = \overline{y}$ for all θ (so that $y^{*'}(\cdot) = 0$) then this condition is satisfied as along as $x^{*\prime}(\cdot) > 0$. Now, suppose that $y^*(\theta) = \overline{y}$ for all θ. We will then show that this is the case. Assuming an interior solution, the first-order condition with respect to x gives:

$$\theta\alpha_1(x^*(\theta), \overline{y})(W + \pi) - 1 - h(\theta)\alpha_1(x^*(\theta), \overline{y})\pi = 0 \tag{11}$$

Note that (11) is exactly the same FOC as in the pure AS environment given in Proposition 1. But, for each θ this FOC has a unique solution and so we must have $x^*(\theta) = x^{AS}(\theta)$. Also note that the condition $\frac{c}{\pi} < \underline{\theta}\alpha_2(x^{AS}(\underline{\theta}), \overline{y})$ implies $x^{AS}(\theta) > 0$ for all θ. Next, since $x^{AS\prime}(\cdot) > 0$, as shown in Proposition 1, the IC condition that we ignored is satisfied when $x^*(\cdot) = x^{AS}(\cdot)$ and $y(\cdot) = \overline{y}$.

Thus far, we have assumed that when the funder chooses $y(\cdot)$ then $y^*(\theta) = \overline{y}$ for all θ. We now show that this is in fact the case. First, see that (11) implies:

$$\theta\alpha_1(0, \overline{y})\pi - 1 + \alpha_1(0, \overline{y})(\theta W - h(\theta)\pi) > 0$$

Condition *P1* means $\theta W - h(\theta)\pi > 0$. Then by differentiating the maximand of the funder's problem with respect to y and evaluating at $x(\cdot) = x^{AS}(\cdot)$, it holds that $y^*(\theta) = \overline{y}$ iff:

$$\theta\alpha_2(x^{AS}(\theta), \overline{y})(W + \pi) - c - h(\theta)\alpha_2(x^{AS}(\theta), \overline{y})\pi \geq 0$$

Rearranging:

$$\theta\alpha_2(x^{AS}(\theta), \overline{y})\pi - c + \alpha_2(x^{AS}(\theta), \overline{y})(\theta W - h(\theta)\pi) \geq 0 \tag{12}$$

By assumption, $\theta\alpha_2(x^{AS}(\theta),\overline{y})\pi - c > 0$, and as already shown, $\theta W - h(\theta)\pi > 0$. Thus, (12) holds with strict inequality, and it is indeed optimal to choose $y^*(\theta) = \overline{y}$ for all θ. But the analysis so far has assumed that the funder is able to choose both x and y; the funder's payoff when the researcher chooses y is bounded from above by his payoff when she may choose y. But under the hypotheses of the proposition: $\theta\alpha_2(x^{AS}(\theta),\overline{y})\pi - c > 0$ for all θ. This implies that when the investment schedule is $x^{AS}(\theta)$ a researcher of any type will indeed choose to exert maximal effort. This means that when y is not contractible, the funder can achieve the upper bound on her payoff by choosing $x^*(\cdot) = x^{AS}(\cdot)$ and $V^*(\cdot) = 0$. This establishes (i).

Next, we show that $g^*(\theta) = g^{AS}(\theta) + c\overline{y}$. Let $U^*(\theta)$ denote the payoff to the researcher when the investment schedule is $x^{AS}(\theta)$ and the researcher chooses $y^* = \overline{y}$. Identical arguments as given in the proof of Proposition 1 may be applied to reveal that IC and IR imply $U^*(\theta) = \int_{\underline{\theta}}^{\theta} \alpha(x^{AS}(t),\overline{y})\pi\, dt$, which is exactly the expression for the researcher's payoff found in the proof of Proposition 1. So, the rent captured by the researcher is the same as in the case where $c = 0$. This implies that for all θ: $g^*(\theta) = g^{AS}(\theta) + c\overline{y}$. This establishes (ii).

To show (iii), first, note that $\underline{\theta}\alpha_2(x^*(\underline{\theta}),\overline{y})\pi - c > 0 \implies \underline{\theta}\alpha(x^*(\underline{\theta}),\overline{y})\pi - c\overline{y} > 0$. Then, $U^*(\underline{\theta}) = 0$ implies:

$$x^*(\underline{\theta}) - g^*(\underline{\theta}) = \underline{\theta}\alpha(x^*(\underline{\theta}),\overline{y})\pi - c\overline{y} > 0$$

Then since $g^*(\theta) = g^{AS}(\theta) + c\overline{y}$ for all θ, this means $g^{*\prime}(\theta) = g^{AS\prime}(\theta) < x^{*\prime}(\theta)$ for all θ; where the inequality follows from Proposition 1. Hence, $g^*(\theta) < x^*(\theta)$ and $g^{*\prime}(\theta) < x^{*\prime}(\theta)$ for all θ.

□

Proof of Proposition 5. The condition $\frac{c}{\pi} \leq \underline{\theta}\alpha(x^{AS}(\theta))$ ensures that the optimal means of funding uses only a grant. Hence,

$$\phi^*(c) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\alpha(x^{AS}(\theta))(W+\pi) - x^{AS}(\theta) - c - h(\theta)\alpha(x^{AS}(\theta))\pi\right] dF(\theta)$$

Appendix B provides the solution to the funder's pure prize problem for any $c \geq 0$. Using this characterization, it follows that

$$\phi^p(c) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\alpha(x^p(\theta))(W+\pi) - x^p(\theta) - c\left[1+\tilde{h}(\theta)\right] - \tilde{h}(\theta)x^p(\theta)\right] dF(\theta)$$

The envelope theorem implies:

$$\frac{\partial \phi^*(c)}{\partial c} - \frac{\partial \phi^p(c)}{\partial c} = \int_{\underline{\theta}}^{\overline{\theta}} \tilde{h}(\theta)\, dF(\theta) > 0$$

□

Proof of Proposition 6. The funder's problem is:

$$\max_{\{x(\cdot),V(\cdot),g(\theta)\}} \int_{\underline{\theta}}^{\overline{\theta}} [y(\cdot)\theta\alpha(x(\theta))(W - V(\theta)) - g(\theta)]dF(\theta) \tag{13}$$

Subject to IC/IR, and where $y(\theta)$ is an indicator function equal to 1 if $\theta\alpha(x(\theta))(V+\pi) - c \geq 0$ and equal to 0 otherwise. If $y(\hat{\theta}) = 1$ then IC requires that $y(\theta) = 1$ for all $\theta > \hat{\theta}$. Also note that $y(\theta) = 1$ if and only if $x(\theta) > 0$. Thus, for any feasible contract, we may assume WLOG that there exists some $\theta^m \in \Theta$ such that $x(\theta), y(\theta) > 0$ for $\theta \geq \theta^m$ and $x(\theta) = y(\theta) = 0$ for $\theta < \theta^*$. To begin, I establish the following lemma.

Lemma 1. *IC and IR imply:* $U'(\theta) \geq \frac{c}{\theta^m}$ *and* $U(\theta) \geq c\left(\frac{\theta-\theta^m}{\underline{\theta}^m}\right)$ *for all* $\theta \in [\theta^m, \overline{\theta}]$.

Proof. In this proof, I focus only on $\theta \geq \theta^m$. IC requires $U'(\theta) = \alpha(x(\theta))(V(\theta)+\pi)$, and since $y(\theta) = 1$ for $\theta > \theta^m$, it must be that $\theta\alpha(x(\theta))(V(\theta)+\pi) \geq c$, or $U'(\theta) \geq \frac{c}{\theta}$ for all $\theta \geq \theta^m$. In particular, this gives $U'(\theta^m) \geq \frac{c}{\theta^m}$. IC also requires:

$$U_{12}(\theta|\theta) = x'(\theta)\alpha'(x(\theta))(V(\theta)+\pi) + \alpha(x(\theta))V'(\theta) \geq 0 \forall \theta \in [\theta^m, \overline{\theta}] \tag{14}$$

and

$$U_1(\theta|\theta) = \theta\Big[x'(\theta)\alpha'(x(\theta))(V(\hat{\theta})+\pi)+\alpha(x(\theta))V'(\theta)\Big]-x'(\theta)+g'(\theta) = 0 \forall\theta \in [\theta^m, \overline{\theta}] \tag{15}$$

Equations (14) and (15) together imply: $x'(\theta) \geq g'(\theta)$. Using the definition of $U(\cdot)$, $U'(\theta) = \alpha(x(\theta))(V(\theta)+\pi)$ may be equivalently expressed: $U'(\theta) = \frac{U(\theta)+c+x(\theta)-g(\theta)}{\theta}$. Using this fact, it follows:

$$U''(\theta) = \frac{\big[U'(\theta)+x'(\theta)-g'(\theta)\big]\theta - \big[U(\theta)+c+x(\theta)-g(\theta)\big]}{\theta^2} = \frac{1}{\theta}\big[x'(\theta)-g'(\theta)\big] \geq 0$$

So, $U'(\theta^m) \geq \frac{c}{\theta^m}$ and $U''(\theta) \geq 0$, implies $U'(\theta) \geq \frac{c}{\theta^m}$ for all $\theta \geq \theta^m$. IR requires $U(\theta^m) \geq 0$. Combining these facts yields $U(\theta) \geq c\left(\frac{\theta-\theta^m}{\theta^m}\right)$ for all $\theta \geq \theta^m$. □

Now consider the contract menu given in the proposition, where for $\theta \geq \theta^m$: $x^*(\cdot) = x^e(\cdot)$, $g^*(\cdot) = x^e(\cdot)$ and $V^*(\cdot) = \frac{c}{\theta^m \alpha(x^e(\theta))} - \pi$. Note that condition *P2* ensures that for all $\theta \in \Theta$, the efficient investment/effort levels satisfy: $x^e(\theta) > 0$ and $y^e(\theta) = 1$; where for each $\theta \in \Theta$, $x^e(\theta)$ solves: $\theta\alpha'(x^e(\theta))(W+\pi)-1 = 0$.

Under the aforementioned contract, it is easily verified that the rent to a type θ researcher that reports $\hat{\theta}$ is: $U(\theta) = c\left(\frac{\theta-\theta^m}{\theta^m}\right)$. Hence, IC is satisfied (trivially), and the funder achieves the bound placed on the researcher's utility, which was found in Lemma 1. For some θ^m, the expected payoff to the funder can then be expressed:

$$\phi(\theta^m) = \int_{\theta^m}^{\overline{\theta}} \left[\theta\alpha(x^e(\theta))(W+\pi) - x^e(\theta) - c - c\left(\frac{\theta-\theta^m}{\theta^m}\right)\right] dF(\theta)$$

I now show that $\phi(\cdot)$ is a single-peaked function of θ^m, and is maximized at some $\theta^* < \overline{\theta}$.

Lemma 2. *$\phi(\cdot)$ is a single-peaked function of θ^m and $\theta^* \equiv \arg\max_{\theta^m\in\Theta}\{\phi(\theta^m)\} <$*

$\overline{\theta}$.

Proof. Let $S(\theta) = \theta\alpha(x^e(\theta))(W+\pi) - x^e(\theta) - c$ denote the maximized total surplus at θ. Note that by condition *P2*, $S(\theta) > 0$ for all θ. Differentiate $\phi(\theta^m)$ with respect to θ^m and re-arrange to obtain:

$$\phi'(\theta^m) = -f(\theta^m)\left[S(\theta^m) - c\tilde{h}(\theta^m)\right]$$

Where $\tilde{h}(\cdot)$ is defined in equation (9). Suppose that at some θ^*: $\phi'(\theta^*) = 0$. Then, $S(\theta^*) = c\tilde{h}(\theta^*)$ and $\phi''(\theta^*) = -f(\theta^*)\left[S'(\theta^*) - c\tilde{h}'(\theta^*)\right] < 0$, which follows since $S'(\cdot) > 0$ (immediate, by the envelope theorem), and $\tilde{h}'(\cdot) < 0$ (by assumption). Thus, $\phi(\cdot)$ is a single-peaked function. Moreover, note that $\phi'(\overline{\theta}) = -f(\overline{\theta})S(\overline{\theta}) < 0$, and hence, $\phi(\cdot)$ attains a maximum at some $\theta^* < \overline{\theta}$, where $\theta^* = \underline{\theta}$ if $S(\underline{\theta}) \geq c\tilde{h}(\underline{\theta})$ and θ^* solves $S(\theta^*) = c\tilde{h}(\theta^*)$ otherwise.

□

I now establish that the contract specified in the Proposition (with θ^* chosen optimally, as outlined in Lemma 2) is indeed optimal for the funder. Under this contract , the funder's payoff can be expressed:

$$\phi^* = \int_{\theta^*}^{\overline{\theta}} \left[\theta\alpha(x^e(\theta))(W+\pi) - x^e(\theta) - c - c\left(\frac{\theta - \theta^*}{\theta^*}\right)\right] dF(\theta)$$

Let $(x(\cdot), V(\cdot), g(\cdot))$ denote any feasible contract (i.e. a contract that satisfies IC/IR and non-negativity), and denote by, ϕ, the payoff to the funder under this contract. Let $\theta^m = \inf\{\theta | x(\theta) > 0\}$. Then:

$$\phi^* = \int_{\theta^*}^{\overline{\theta}} \left[\theta\alpha(x^e(\theta))(W+\pi) - x^e(\theta) - c - c\left(\frac{\theta - \theta^*}{\theta^*}\right)\right] dF(\theta)$$

$$\geq \int_{\theta^m}^{\overline{\theta}} \left[\theta\alpha(x^e(\theta))(W+\pi) - x^e(\theta) - c - c\left(\frac{\theta - \theta^m}{\theta^m}\right)\right] dF(\theta)$$

$$\geq \int_{\theta^m}^{\overline{\theta}} \left[\theta\alpha(x(\theta))(W+\pi) - x(\theta) - c - U(\theta)\right] dF(\theta) = \phi$$

The first inequality follows from the definition of θ^*. The second inequality follows from the definition of x^e, and since Lemma 1 implies $U(\theta) \geq c\left(\frac{\theta-\theta^m}{\theta^m}\right)$.
□

Proof of Proposition 7. We first establish part (i). Differentiating (6) with respect to π we obtain:

$$\frac{\partial x^*(\theta)}{\partial \pi} = -\frac{\alpha_1(x^*(\theta), \overline{y})(\theta - h(\theta))}{\alpha_{11}(x^*(\theta), \overline{y})[\theta(W+\pi) - h(\theta)\pi]}$$

The denominator is strictly negative, and so $\frac{\partial x^*(\theta)}{\partial \pi} > 0$ if and only if $\theta > h(\theta)$. We now establish part (ii). Let $\phi^*(\pi)$ denote the maximized payoff to the funder. Using the expression for the funder's payoff given in (4) and applying the envelope theorem, it holds that:

$$\frac{\partial \phi^*(\pi)}{\partial \pi} = \int_{\underline{\theta}}^{\overline{\theta}} \alpha(x^*(\theta), \overline{y})\,(\theta - h(\theta))\, f(\theta)\, d\theta$$

Note that $[\theta - h(\theta)]$ is strictly increasing in θ with $\overline{\theta} - h(\overline{\theta}) = \overline{\theta} > 0$. So there are two cases to consider. First, if $\theta - h(\theta) > 0$ for all $\theta \in \Theta$ then the result follows immediately. The second possibility is that, for some $\theta^* \in (\underline{\theta}, \overline{\theta})$: $\theta - h(\theta) < 0$ for $\theta < \theta^*$, and $\theta - h(\theta) > 0$ for $\theta > \theta^*$. In this case, using the fact that $x(\cdot)$ is increasing, it follows:

$$\frac{\partial \phi^*(\pi)}{\partial \pi} > \alpha(x^*(\theta^*), \overline{y}) \int_{\underline{\theta}}^{\overline{\theta}} (\theta - h(\theta))\, f(\theta)\, d\theta = \alpha(x(\theta^*))\underline{\theta} \geq 0$$

□

Proof of Proposition 8. The Lagrangian for the funder's problem is

$$\mathcal{L} = \tilde{\theta}\alpha(x,y)(W - V) - \lambda_1 \left[x + cy - \tilde{\theta}\alpha(x,y)(V+\pi)\right] - \lambda_2 \left[c - \tilde{\theta}\alpha_2(x,y)(V+\pi)\right]$$

Where λ_1 and λ_2 are the multipliers associated with the constraints given in equations (7) and (8), respectively. Note that the researcher's effort-choice problem is strictly concave in y for fixed x and V. This ensures that the first-order approach employed here is valid. The first-order conditions (FOCs) with respect to x and y are

$$\frac{\partial \mathcal{L}}{\partial x} = \tilde{\theta}\alpha_1(x^*, y^*)(W - V^*) - \lambda_1^* \left[1 - \tilde{\theta}\alpha_1(x^*, y^*)(V^* + \pi)\right] + \lambda_2^* \left[\alpha_{12}(x^*, y^*)(V^* + \pi)\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \tilde{\theta}\alpha_2(x^*, y^*)(W - V^*) + \lambda_2^* \left[\alpha_{22}(x^*, y^*)(V^* + \pi)\right] = 0$$

Optimality requires $W > V^*$ and hence the FOC with respect to y gives $\lambda_2^* > 0$. Optimality also requires the researcher's IR constraint to bind, and so the complementary slackness condition with respect to the IR constraint implies $\lambda_1^* > 0$. These facts, along with the FOC with respect to x, imply $1 - \tilde{\theta}\alpha_1(x^*, y^*)(V^* + \pi) > 0$. Now, evaluating equation (7) at the optimal solutions (noting that (7) must hold with equality), and differentiating with respect to π gives:

$$\frac{\partial x^*}{\partial \pi} \left[1 - \tilde{\theta}\alpha_1(x^*, y^*))(V^* + \pi)\right] - \tilde{\theta}\alpha(x^*, y^*) \left[\frac{\partial V^*}{\partial \pi} + 1\right] = 0$$

The first term in square brackets above is positive, and so this expression implies sign $\left(\frac{\partial x^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial V^*}{\partial \pi} + 1\right)$. Following the same procedure for equation (8) yields:

$$\alpha_{12}(x^*, y^*)(V^* + \pi)\frac{\partial x^*}{\partial \pi} + \alpha_{22}(x^*, y^*)(V^* + \pi)\frac{\partial y^*}{\partial \pi} + \alpha_2(x^*, y^*)\left[\frac{\partial V^*}{\partial \pi} + 1\right] = 0$$

Since sign $\left(\frac{\partial x^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial V^*}{\partial \pi} + 1\right)$ the expression above implies sign $\left(\frac{\partial y^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial x^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial V^*}{\partial \pi} + 1\right)$. We now show that these derivatives must all be positive.

Equation (7) implies $V = \frac{x+cy}{\tilde{\theta}\alpha(x,y)} - \pi$. Plugging this into the maximand of the funder's problem and into the constraint in equation (8) yields the following equivalent problem for the funder:

$$\max_{x,y \geq 0} \left\{\phi(x, y, \pi) \equiv \tilde{\theta}\alpha(x, y)(W + \pi) - x - cy\right\}$$

Subject to the constraint:

$$\alpha(x, y)c - \alpha_2(x, y)(x + cy) = 0$$

It is clear that the maximand in the funder's problem, $\phi(x, y, \pi)$, is convex in π for fixed x and y. Moreover, the constraint set is independent from π. This means that $\phi^*(\pi)$ – the maximized payoff to the funder – is convex in π. By the envelope theorem: $\frac{\partial \phi^*(\pi)}{\partial \pi} = \tilde{\theta}\alpha(x^*, y^*) > 0$. And hence

$$\frac{\partial^2 \phi^*(\pi)}{\partial \pi^2} = \tilde{\theta}\alpha_1(x^*, y^*)\frac{\partial x^*}{\partial \pi} + \tilde{\theta}\alpha_2(x^*, y^*)\frac{\partial y^*}{\partial \pi} \geq 0$$

But since sign $\left(\frac{\partial x^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial y^*}{\partial \pi}\right) = \text{sign}\left(\frac{\partial V^*}{\partial \pi} + 1\right)$, we must have $\frac{\partial x^*}{\partial \pi} \geq 0$, $\frac{\partial y^*}{\partial \pi} \geq 0$, and $\frac{\partial V^*}{\partial \pi} + 1 \geq 0$. The stated comparative statics results with respect to $\tilde{\theta}$ follow from an analogous procedure, and so I omit the arguments.

□

Appendix B: The Pure Prize Problem

Assume the binary effort specification as outlined in section 6. The case with AS only corresponds to an environment in which $c = 0$ and $y = 1$.

First note that in this setting, the payoff to a type θ researcher who reports $\hat{\theta}$ is:

$$U(\hat{\theta}|\theta) = y(\theta, x(\hat{\theta}), V(\hat{\theta}) + \pi)(\theta\alpha(x(\hat{\theta}))(V(\hat{\theta}) + \pi) - c) - x(\hat{\theta})$$

As outlined in the proof of Proposition 1, IC and IR requires for all $\theta, \hat{\theta} \in \Theta$: $U'(\theta) = U_1(\theta|\theta)$, $U_{12}(\hat{\theta}|\theta) \geq 0$, $U(\theta) \geq 0$. These conditions may be restated: $U'(\theta) = y(\cdot)\alpha(x(\theta))(V(\theta) + \pi)$, $U(\underline{\theta}) \geq 0$ and $x'(\theta) \geq 0$. Note that $U(\theta) \geq 0$ means

$$y(\cdot)(\theta\alpha(x(\theta))(V(\theta) + \pi) - c) \geq x(\theta)$$

Hence, the IR constraint requires $\theta\alpha(x(\theta))(V(\theta)+\pi)-c \geq 0$, which ensures that any researcher given a positive level of investment will exert effort. So, without loss of generality we will set $y = 1$ for all types given a positive investment recommendation. Then, a simple substitution into the IC constraint yields:

$$U'(\theta) = \frac{U(\theta) + x(\theta) + c}{\theta}$$

Using the fact that $U(\underline{\theta}) = 0$ and solving this differential equation we obtain

$$U(\theta) = \theta \int_{\underline{\theta}}^{\theta} \frac{x(t) + c}{t^2} \, dF(t)$$

Integrating by parts:

$$\int_{\underline{\theta}}^{\overline{\theta}} U(\theta) \, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \tilde{h}(\theta)(x(\theta) + c) \, dF(\theta)$$

Where $\tilde{h}$ is defined in equation (9). Substituting $U(\theta)$ into the funder's problem for $V(\theta)$, the problem faced by the funder may be expressed:

$$\max_{x(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\alpha(x(\theta))(W+\pi) - x(\theta) - \tilde{h}(\theta)x(\theta) - c(1+\tilde{h}(\theta))\right] dF(\theta)$$

Subject to the non negativity constraint $x(\theta) \geq 0$ and the IC constraint $x'(\theta) \geq 0$. Note that concavity of α implies that the maximand above is strictly concave in x at each θ. Maximization over this expression yields the necessary and sufficient first-order condition:

$$\theta\alpha'(x^p(\theta))(W+\pi) - 1 - \tilde{h}(\theta) = 0$$

Differentiating the first-order condition with respect to θ we see that $\tilde{h}'(\theta) < 0$ is sufficient to ensure $x^p(\theta)$ is strictly decreasing, as required by IC.